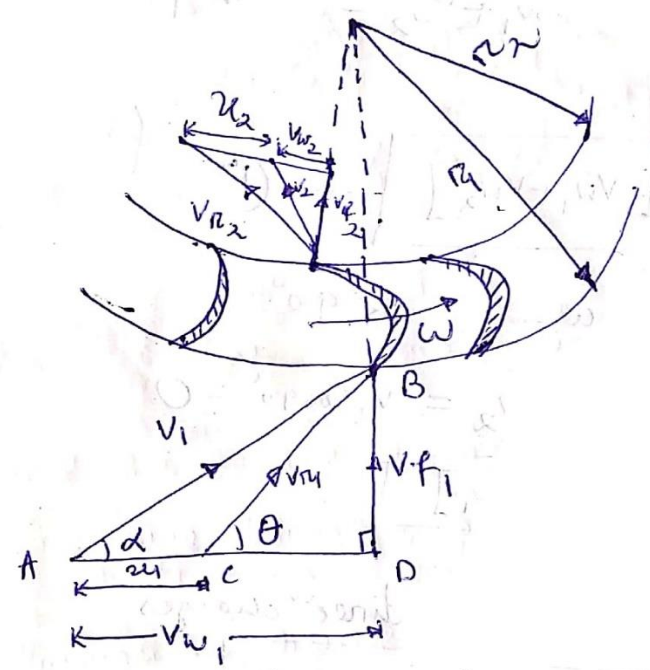
Force exerted on a series of Radial Curved Vanes. (series of vane mounted on wheel.)



r_1 = radius of wheel at inlet of vane

r_2 = radius of wheel at out-let of vane

ω = angular velocity of wheel

u_1 = Velocity of vane at inlet $= r_1 \omega$

u_2 = " " " outlet $= r_2 \omega$

$\dot{m}$ = mass of water striking per secon

$= \rho a V_1$

momentum at inlet of vane per second

$= \dot{m} \times V_1 \cos\alpha = \rho a V_1 V_{w_1}$

momentum at out-let of vane per second

$= \dot{m} \times (-V_2 \cos\beta) = \rho a V_1 \times (-V_{w_2})$

Angular momentum at inlet perseond

$= $ momentum at inlet $\times$ radius

$= \rho a V_1 V_{w_1} \times r_1$

Angular momentum at out-let per second

$= \rho a V_1 (-V_{w_2}) \times r_2$

Torque exerted by water on wheel

$T =$ rate of change of angular momentum

$=$ Initial angular momentum $-$ Final angular moment

$= \rho a V_1 V_{w_1} r_1 - (-\rho a V_1 V_{w_2} \times r_2)$

$= \rho a V_1 [V_{w_1} r_1 + V_{w_2} r_2]$

Workdone per second on wheel

$= T \times \omega$

$= \rho a V_1 [V_{w_1} r_1 + V_{w_2} r_2] \times \omega$

$= \rho a V_1 [V_{w_1} r_1 \omega + V_{w_2} r_2 \omega]$

$$W/s = \rho a V_1 [V_{w_1} u_1 + V_{w_2} u_2]$$

If $\beta = 90^\circ$, $V_{w_2} = 0$

$$W/s = \rho a V_1 V_{w_1} u_1$$

If $\beta > 90^\circ$, $W/s = \rho a V_1 [V_{w_1} u_1 - V_{w_2} u_2]$

general expression of workdone/second

$$W/s = \rho a V_1 [V_{w_1} u_1 \pm V_{w_2} u_2]$$

$$\eta = \frac{W/sec}{K.E/sec} = \frac{\rho a V_1 [V_{w_1} u_1 \pm V_{w_2} u_2]}{\frac{1}{2}(\rho a V_1) V_1^2}$$

$$\boxed{\eta = \frac{2[V_{w_1} u_1 \pm V_{w_2} u_2]}{V_1^2}}$$

Problem 17.11 *A jet of water of diameter 10 cm strikes a flat plate normally with a velocity of 15 m/s. The plate is moving with a velocity of 6 m/s in the direction of the jet and away from the jet. Find:*

(i) the force exerted by the jet on the plate

(ii) work done by the jet on the plate per second.

Solution. Given :

Diameter of the jet, $d = 10 \text{ cm} = 0.1 \text{ m}$

$\therefore$ Area, $a = \frac{\pi}{4} d^2 = \frac{\pi}{4} (.1)^2 = .007854 \text{ m}^2$

Velocity of jet, $V = 15 \text{ m/s}$

Velocity of the plate, $u = 6 \text{ m/s}$.

(*i*) The force exerted by the jet on a moving flat vertical plate is given by equation (17.11),

$$F_x = \rho a (V - u)^2$$

$$= 1000 \times .007854 \times (15 - 6)^2 \text{ N} = \mathbf{636.17 \text{ N}}. \textbf{ Ans.}$$

(*ii*) Work done per second by the jet

$$= F_x \times u = 636.17 \times 6 = \mathbf{3817.02 \text{ Nm/s}}. \textbf{ Ans.}$$

Problem 17.1 *Find the force exerted by a jet of water of diameter 75 mm on a stationary flat plate, when the jet strikes the plate normally with velocity of 20 m/s.*

Solution. Given :

Diameter of jet, $d = 75 \text{ mm} = 0.075 \text{ m}$

$\therefore$ Area, $a = \frac{\pi}{4} d^2 = \frac{\pi}{4} (.075)^2 = .004417 \text{ m}^2$

Velocity of jet, $V = 20 \text{ m/s.}$

The force exerted by the jet of water on a stationary vertical plate is given by equation (17.1) as

$$F = \rho a V^2 \quad \text{where } \rho = 1000 \text{ kg/m}^3$$

$\therefore$ $F = 1000 \times .004417 \times 20^2 \text{ N} = $ **1766.8 N. Ans.**

Problem 17.2 *Water is flowing through a pipe at the end of which a nozzle is fitted. The diameter of the nozzle is 100 mm and the head of water at the centre nozzle is 100 m. Find the force exerted by the jet of water on a fixed vertical plate. The co-efficient of velocity is given as 0.95.*

Solution. Given :

Diameter of nozzle, $d = 100 \text{ mm} = 0.1 \text{ m}$

Head of water, $H = 100 \text{ m}$

Co-efficient of velocity, $C_v = 0.95$

Area of nozzle, $a = \frac{\pi}{4} (.1)^2 = .007854 \text{ m}^2$

Theoretical velocity of jet of water is given as

$$V_{th} = \sqrt{2gH} = \sqrt{2 \times 9.81 \times 100} = 44.294 \text{ m/s}$$

But $C_v = \frac{\text{Actual velocity}}{\text{Theoretical velocity}}$

$\therefore$ Actual velocity of jet of water, $V = C_v \times V_{th} = 0.95 \times 44.294 = 42.08 \text{ m/s.}$

Force on a fixed vertical plate is given by equation (17.1) as

$$F = \rho a V^2 = 1000 \times .007854 \times 42.08^2 \quad (\because \text{ In S.I. units } \rho \text{ for water} = 1000 \text{ kg/m}^3)$$

$= 13907.2 \text{ N} = $ **13.9 kN. Ans.**

Problem 17.12 (A) *A nozzle of 50 mm diameter delivers a stream of water at 20 m/s perpendicular to a plate that moves away from the jet at 5 m/s. Find :*

(i) the force on the plate,

(ii) the work done, and

(iii) the efficiency of jet. (J.N.T.U., Hyderabad S 2002)

Solution. Given :

Dia. of jet $= 50 \text{ mm} = 0.05 \text{ m}$

$\therefore$ Area, $a = \frac{\pi}{4}(0.05^2) = 0.0019635 \text{ m}^2$

Velocity of jet, $V = 20$ m/s, Velocity of plate, $u = 5$ m/s

(*i*) The force on the plate is given by equation (17.11) as,

$$F_x = \rho a (V - u)^2$$

$$= 1000 \times 0.0019635 \times (20 - 5)^2 = \textbf{441.78 N. Ans.}$$

(*ii*) The work done by the jet

$$= F_x \times u = 441.78 \times 5 = \textbf{2208.9 Nm/s. Ans.}$$

(*iii*) The efficiency of the jet, $\eta = \dfrac{\text{Output of jet}}{\text{Input of jet}}$

$$= \frac{\text{Work done/s}}{\text{K.E. of jet/s}} = \frac{F_x \times u}{\frac{1}{2} m V^2}$$

$$= \frac{F_x \times u}{\frac{1}{2}(\rho A V) \times V^2}$$

$$= \frac{2208.9}{\frac{1}{2}(1000 \times 0.0019635 \times 20) \times 20^2} = \frac{2208.9}{6540}$$

$= 0.3377 = \textbf{33.77\%. Ans.}$

Problem 17.24 *If in Problem 17.23, the jet of water instead of striking a single plate, strikes a series of curved vanes, find for the data given Problem 17.23,*

(i) Force exerted by the jet on the vane in the direction of motion,

(ii) Power exerted on the vane, and

(iii) Efficiency of the vane.

Solution. Given :

From Problem 17.23, $V_1 = 15$ m/s, $u = u_1 = u_2 = 5$m/s,

$\alpha = 0$, $a = .007854 \text{ m}^2$

$\phi = 45°$, $V_{w_1} = 15$ m/s and $V_{w_2} = 2.07$ m/s.

For the series of vanes, mass of water striking per second

= Mass of water coming out from nozzle

$= \rho a V_1 = 1000 \times .007854 \times 15 = 117.72$

(*i*) Force exerted by the jet on the vane in the direction of motion

$$F_x = \rho a V_1 [V_{w_1} + V_{w_2}] = 117.72 [15 + 2.07] = \mathbf{2009.5 \text{ N. Ans.}}$$

(*ii*) Power of the vane in kW

$$= \frac{\text{Work done per second}}{1000} = \frac{F_x \times u}{1000} \text{kW} = \frac{2009.5 \times 5}{1000}$$

$= \mathbf{10.05 \text{ kW. Ans.}}$

(*iii*) Efficiency,

$$\eta = \frac{\text{Work done per second}}{\frac{1}{2}(\text{mass of water per sec}) \times V_1^2}$$

$$= \frac{2009.5 \times 5.0}{\frac{1}{2} \times 117.72 \times 15^2} = 0.7586 \text{ or } \mathbf{75.86\%. \text{ Ans.}}$$

Problem 17.25 *A jet of water having a velocity of 35 m/s impinges on a series of vanes moving with a velocity of 20 m/s. The jet makes an angle of 30° to the direction of motion of vanes when entering and leaves at an angle of 120°. Draw the triangles of velocities at inlet and outlet and find :*

(a) the angles of vanes tips so that water enters and leaves without shock,

(b) the work done per unit weight of water entering the vanes, and

(c) the efficiency.

Solution. Given :

Velocity of jet, $V_1 = 35$ m/s

Velocity of vane, $u_1 = u_2 = 20$ m/s

Angle of jet at inlet, $\alpha = 30°$

Angle made by the jet at outlet with the direction of motion of vanes = 120°

∴ Angle $\beta = 180° - 120° = 60°$

(*a*) *Angle of vanes tips.*

From inlet velocity triangle

$$V_{w_1} = V_1 \cos\alpha = 35 \cos 30° = 30.31 \text{ m/s}$$

$$V_{f_1} = V_1 \sin\alpha = 35 \sin 30° = 17.50 \text{ m/s}$$

$$\tan\theta = \frac{V_{f_1}}{V_{w_1} - u_1} = \frac{17.50}{30.31 - 20} = 1.697$$

∴ $$\theta = \tan^{-1} 1.697 = \mathbf{60°.\ Ans.}$$

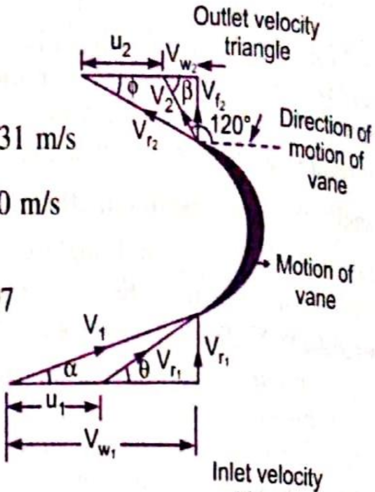


Fig. 17.23(*a*)

By sine rule, $$\frac{V_{r_1}}{\sin 90°} = \frac{V_{f_1}}{\sin\theta} \quad \text{or} \quad \frac{V_{r_1}}{1} = \frac{17.50}{\sin 60°}$$

∴ $$V_{r_1} = \frac{17.50}{.866} = 20.25 \text{ m/s.}$$

Now, $$V_{r_2} = V_{r_1} = 20.25 \text{ m/s}$$

From outlet velocity triangle, by sine rule

$$\frac{V_{r_2}}{\sin 120°} = \frac{u_2}{\sin(60° - \phi)} \quad \text{or} \quad \frac{20.25}{0.886} = \frac{20}{\sin(60° - \phi)}$$

∴ $$\sin(60° - \phi) = \frac{20 \times 0.866}{20.25} = 0.855 = \sin(58.75°)$$

$$60° - \phi = 58.75°$$

∴ $$\phi = 60° - 58.75° = \mathbf{1.25°.\ Ans.}$$

(*b*) *Work done per unit weight of water entering* $= \frac{1}{g}(V_{w_1} + V_{w_2}) \times u_1$ ...(*i*)

$$V_{w_1} = 30.31 \text{ m/s and } u_1 = 30 \text{ m/s}$$

The value of V_{w_2} is obtained from outlet velocity triangle

$$V_{w_2} = V_{r_2} \cos\phi - u_2 = 20.25 \cos 1.25° - 20.0 = 0.24 \text{ m/s}$$

∴ Work done/unit weight $$= \frac{1}{9.81}[30.31 + 0.24] \times 20 = \mathbf{62.28\ Nm/N.\ Ans.}$$

(*c*) *Efficiency* $$= \frac{\text{Work done per kg}}{\text{Energy supplied per kg}}$$

$$= \frac{62.28}{\frac{V_1^2}{2g}} = \frac{62.28 \times 2 \times 9.81}{35 \times 35} = \mathbf{99.74\%\ Ans.}$$

Problem 17.26 *A jet of water having a velocity of 30 m/s strikes a series of radial curved vanes mounted on a wheel which is rotating at 200 r.p.m. The jet makes an angle of 20° with the tangent to the wheel at inlet and leaves the wheel with a velocity of 5 m/s at an angle of 130° to the tangent to the wheel at outlet. Water is flowing from outward in a radial direction. The outer and inner radii of the wheel are 0.5 m and 0.25 m respectively. Determine :*

(i) Vane angles at inlet and outlet, *(ii) Work done per unit weight of water, and*

(iii) Efficiency of the wheel.

Solution. Given :

Velocity of jet, $V_1 = 30$ m/s

Speed of wheel, $N = 200$ r.p.m.

$\therefore$ Angular speed, $\omega = \dfrac{2\pi N}{60} = \dfrac{2\pi \times 200}{60} = 20.94$ rad/s

Angle of jet at inlet, $\alpha = 20°$

Velocity of jet at outlet, $V_2 = 5$ m/s

Angle made by the jet at outlet with the tangent to wheel = 130°

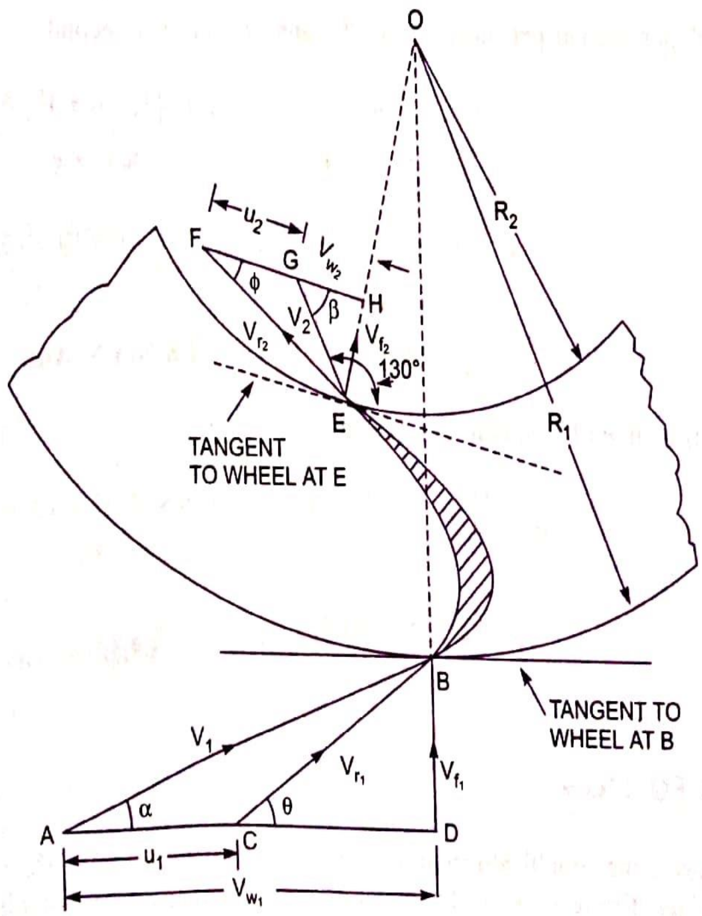
$\therefore$ Angle, $\beta = 180° - 130° = 50°$

Outer radius, $R_1 = 0.5$ m

Inner radius, $R_2 = 0.25$ m

$\therefore$ Velocity $u_1 = \omega \times R_1 = 20.94 \times 0.5 = 10.47$ m/s

And $u_2 = \omega \times R_2 = 20.94 \times 0.25 = 5.235$ m/s.



(*i*) **Vane angles** at inlet and outlet means the angle made by the relative velocities V_{r_1} and V_{r_2} *i.e.*, angle θ and ϕ.

From ΔABD,
$$V_{w_1} = V_1 \cos\alpha = 30 \times \cos 20° = 28.19 \text{ m/s}$$
$$V_{f_1} = V_1 \sin\alpha = 30 \times \sin 20° = 10.26 \text{ m/s}$$

In ΔCBD,
$$\tan\theta = \frac{BD}{CD} = \frac{V_{f_1}}{AD - AC} = \frac{10.26}{V_{w_1} - u_1} = \frac{10.26}{28.19 - 10.47} = 0.579 = \tan 30.07$$

$\therefore$ $\theta = 30.07°$ or **30° 4.2′. Ans.**

From outlet velocity Δ,
$$V_{w_2} = V_2 \cos\beta = 5 \times \cos 50° = 3.214 \text{ m/s}$$
$$V_{f_2} = V_2 \times \sin\beta = 5 \sin 50° = 3.83 \text{ m/s}$$

In ΔEFH,
$$\tan\phi = \frac{V_{f_2}}{u_2 + V_{w_2}} = \frac{3.83}{5.235 + 3.214} = 0.453 = \tan 24.385°$$

$\therefore$ $\phi = 24.385°$ or **24° 23.1′. Ans.**

(*ii*) Work done per second by water is given by equation (17.26)

$$= \rho a V_1 \left[V_{w_1} u_1 + V_{w_2} u_2\right]$$

(+ ve sign is taken as β is acute angle in Fig.17.24)

$\therefore$ Work done* per second per unit weight of water striking per second

$$= \frac{\rho a V_1\left[V_{w_1} u_1 + V_{w_2} u_2\right]}{\text{Weight of water/s}} = \frac{\rho a V_1\left[V_{w_1} u_1 + V_{w_2} u_2\right]}{\rho a V_1 \times g}$$

$$= \frac{1}{g}\left[V_{w_1} u_1 + V_{w_2} u_2\right] \text{ Nm/N} = \frac{1}{9.81}\left[28.19 \times 10.47 + 3.214 \times 5.235\right]$$

$$= \frac{1}{9.81}\left[295.15 + 16.82\right] = \mathbf{31.8 \text{ Nm/N. Ans.}}$$

(*iii*) Efficiency, η is given by equation (17.28) as

$$\eta = \frac{2\left[V_{w_1} u_1 + V_{w_2} u_2\right]}{V_1^2} = \frac{2\left[28.19 \times 10.47 + 3.214 \times 5.235\right]}{30^2}$$

$$= \frac{2\left[295.15 + 16.82\right]}{30 \times 30} = 0.6932 \text{ or } \mathbf{69.32\%. \text{ Ans.}}$$