CENTROID AND MOMENT OF INERTIA

Introduction :-

A body consists of a number of particles and earth attracts each particle towards its centre. This force of attraction is directly proportional to the mass of the particle which acts vertically downwards called **Weight** of body.

$$F \propto m$$

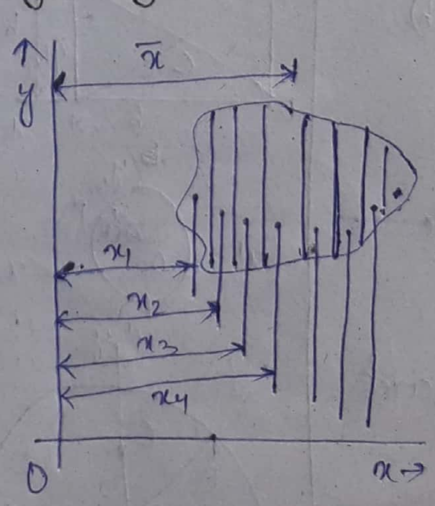
As the distance between the different particles of a body and the centre of the earth is same (assumption), these forces may be taken to act along parallel line. The point where resultant of all such parallel forces act is called centre of gravity.

Centre of gravity :- Centre of gravity is defined as the point through which the whole weight of the body acts, irrespective of its position.

→ For every body, there is only one centre of gravity.

Centroid :- The plane figures (like triangle, circle etc.) have only areas, but no mass. The centre of area of such figures is known as centroid.

Position of Centroid / Centre of gravity :-



Location of centroid $(\bar{x}, \bar{y})$ can be found out by calculating moment of small areas $a_1, a_2, a_3 \ldots, a_n$ which constitute area A about an axis.

∴ The moment of all small strips about y-axis,

$$= a_1x_1 + a_2x_2 + a_3x_3 + \ldots + a_nx_n$$

$$= \Sigma ax \quad -(1)$$

Moment of whole area $A = A\bar{x}$ — (2)

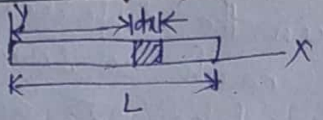
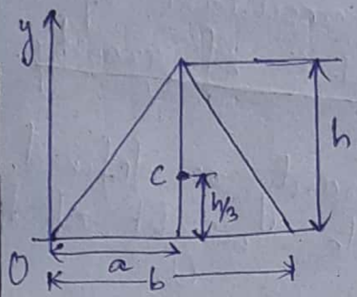
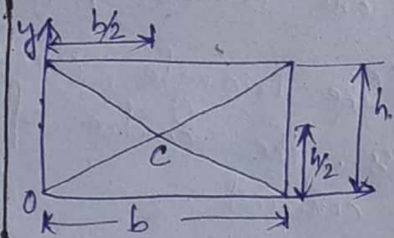
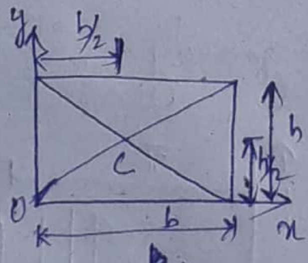
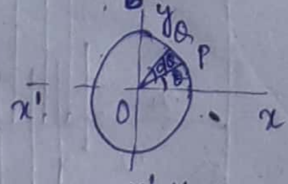
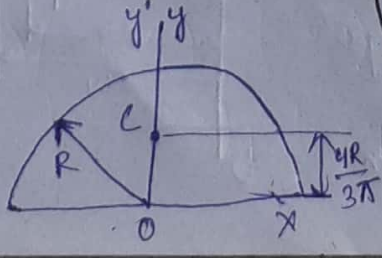
$\Rightarrow \Sigma ax = A\bar{x} \Rightarrow \boxed{\bar{x} = \dfrac{\Sigma ax}{A}}$ — (3)

Similarly, moment about y-axis,

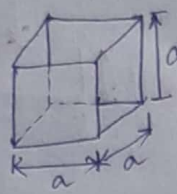
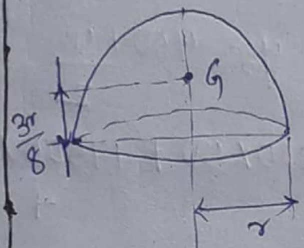
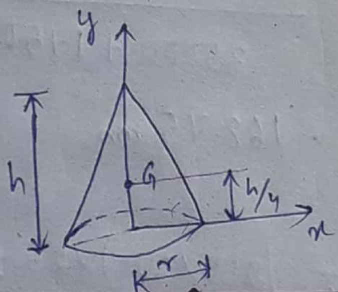
$\Rightarrow \boxed{\bar{y} = \dfrac{\Sigma ay}{A}}$ — (4)

where Σay = Algebraic sum of moment about x-axis.

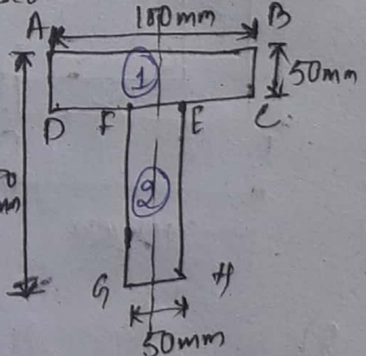
Centroids of various geometrical shapes

Description	Shape	Area	$\bar{x}$	$\bar{y}$
Rod		L (if width is neglected)	$\bar{x} = L/2$	0 (since width = 0)
Triangle		$\frac{1}{2} bh$	$\frac{1}{3}(a+b)$	$h/3$
Rectangle		bh	$b/2$	$h/2$
Square ($a=b=h$)		a^2	$a/2$	$a/2$
Circle		πr^2	At centre = 0	At centre = 0
Semicircle		$\frac{\pi r^2}{2}$	0	$= \frac{4R}{3\pi} = 0.424R$

Centre of gravity for various solids :-

Description	Shape	Volume	$\bar{y}$ (from base)
Cube		a^3	$a/2$ (from all sides).
Sphere		$\frac{4}{3}\pi r^3$	$d/2$ d = diameter
Hemisphere		$\frac{2}{3}\pi r^3$	$\frac{3r}{8}$
Right circular solid cone.		$\frac{1}{3}\pi r^2 h$.	$h/4$

Q) Find centre of gravity of a T-section 100X200X50mm



A - T. section is symmetrical about y-axis.

$$\bar{y} = \frac{a_1 y_1 + a_2 y_2}{a_1 + a_2}$$

In ①

$a_1 = 100 \times 50 = 5000\ mm^2$

$y_1 = \left(200 - \frac{50}{2}\right) = 175\ mm.$

In ②

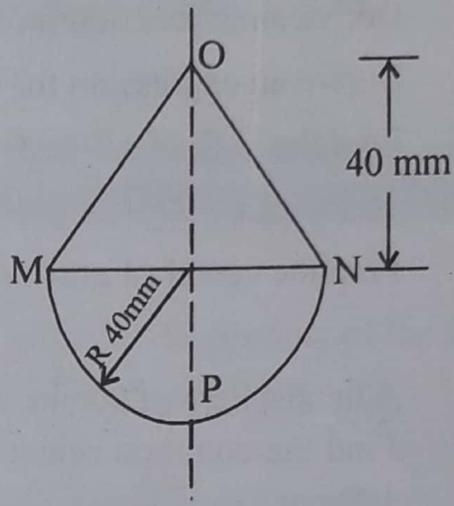
$a_2 = 150 \times 50 = 7500\ mm^2 \quad y_2 = \frac{150}{2} = 75\ mm.$

$$\bar{y} = \frac{a_1 y_1 + a_2 y_2}{a_1 + a_2} = \frac{5000 \times 175 + 7500 \times 75}{5000 + 7500} = 115\ mm$$

Example – 6 : ***A body consists of a right circular solid cone of height 40mm and radius 40mm placed on a solid hemisphere of radius 30mm of the same material. Determine the position of the centre of gravity of the body.***

Solution :

The composite solid is symmetrical about y-axis. Let point P be the point of reference.



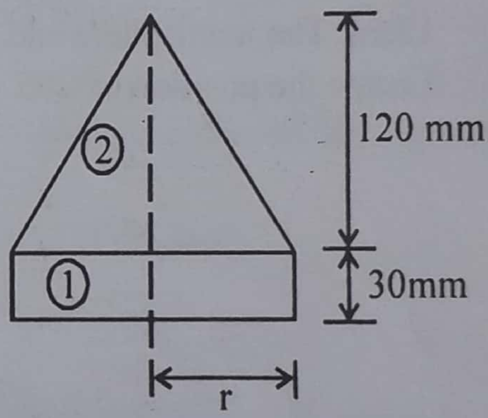
$$\bar{y} = \frac{v_1 y_1 + v_2 y_2}{v_1 + v_2}$$

$$= \frac{\left(\frac{2\pi}{3} \times R^3 \times \frac{5R}{8}\right) + \left(\frac{\pi}{3} \times R^2 \times h\right) \times \left(40 + \frac{40}{4}\right)}{\frac{2\pi}{3} \times R^3 + \frac{\pi}{3} \times R^2 \times h}$$

$$= \frac{\frac{5R^2}{12} + h \times 50}{2R + h} = \frac{\frac{5 \times 40^2}{12} + 40 \times 50}{2 \times 40 + 40}$$

$= 22.23$mm.

Example – 7 : ***A right circular cone of height 120mm is placed on a cylinder of height 30mm. The bases of both the sides are equal. Determine the distance of C.G of the composite solid from the bottom base.***



Solution :

The composite solid is symmetrial about Y-axis.
Let the base of cylinder be the plane of reference.

$$\bar{y} = \frac{v_1 y_1 + v_2 y_2}{v_1 + v_2}$$

$$= \frac{\left(\pi r^2 \times 30\right) \times \frac{30}{2} + \left(\frac{\pi}{3} \times r^2 \times 120\right) \times \left(30 + \frac{120}{4}\right)}{\pi r^2 \times 30 + \frac{\pi}{3} r^2 \times 120}$$

$$= \frac{30 \times 15 + \frac{150}{3} \times \left(\frac{120}{4} + 30\right)}{30 + \frac{120}{3}} = \frac{450 + 2400}{30 + 40} = 40.71\text{mm.}$$